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Freejet Terminal Shocks

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Introduction

THE problem of interest here is that of expanding a two-dimensional gas jet into a quiescent medium. The jets studied are highly underexpanded at their entrance into this medium and subsequently form a classical freejet plume with its attendant barrel and terminal shock waves. The objective of the present study was to provide a suitable means of predicting the terminal shock's position relative to the jet exit plane for jet nozzle Mach numbers equal to or greater than one. Motivation for such an effort has come principally from the field of jet interaction controls where experimental and theoretical studies have indicated that the terminal shock height is a significant scale length. Hence, a definite need exists for a simple model of the two-dimensional freejet (cf., Ref. 1 for further discussion of this point).

Experimental Apparatus and Results

The test apparatus was a flat plate upon which was mounted two-glass-ported side plates for viewing the two-dimensional freejet. The jet issued from a 6-in. span sonic nozzle, flush mounted in the flat plate surface. This entire assembly was installed in a wind-tunnel test cell which served as a controlled pressure reservoir; the reservoir pressures P_∞ being set at either 2 psia or 5 psia for these tests. Three sonic jet nozzles (nominal throat widths of 0.005, 0.020, and 0.030 in.) were tested for a range of jet stagnation pressures which varied from 0–1200 psia, the jet stagnation temperature being constant at 520°R. The flow rate of the jet supply air was measured independently with a standard ASME orifice plate flow meter.

Experimentally observed values of the centerline jet-plume shock height h_s (nondimensionalized by the effective nozzle-exit throat width b_e) are presented in Fig. 1. Also shown are some recent data from Sheeran and Dosanjh[2] who tested weak jets over a limited range of back pressures. Both sets of data are well represented by the relation§

$$h_s/b_e = P_e/P_b \qquad (1)$$

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§ The jet-plume back pressure P_b was monitored with a pitot tube placed well above the plume, and in all cases was found to be equal to the reservoir pressure P_∞ to within the accuracy of the instrumentation.

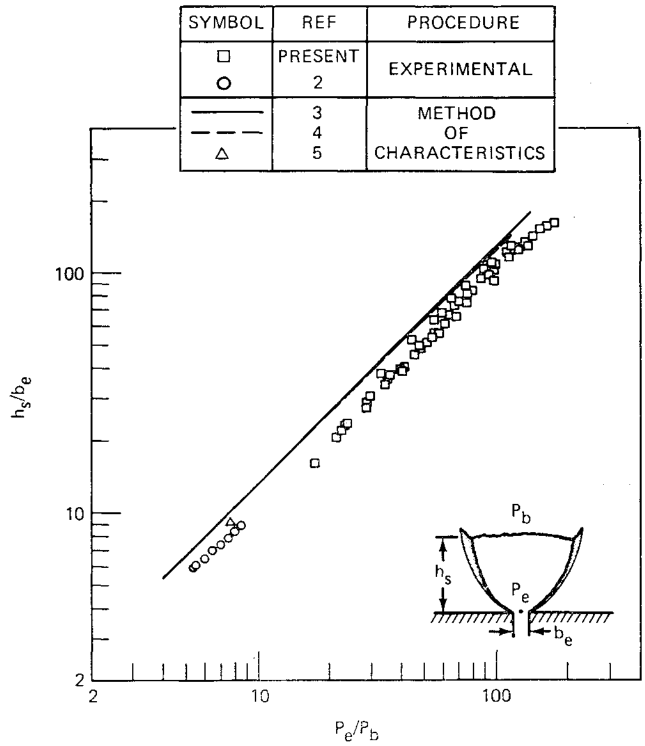

Fig. 1 Two-dimensional freejet terminal shock heights for sonic jets.

To the best of the authors' knowledge, Fig. 1 represents the totality of experimental two-dimensional freejet terminal shock data.

Discussion of Results

One of the principal goals of the present effort was to test the validity of presently available analytical models for the two-dimensional freejet problem. The more sophisticated models employ the method of characteristics to compute the freejet interior flow and thereafter determine the position of the jet shock by either assuming the terminal-shock back pressure, P_b, to be P_∞, or by making some statement about the triple point configuration at the plume termination. The first of these assumptions has more or less stood the test of time for the axisymmetric problem and its application to the two-dimensional problem seems quite natural. Comparison of the solutions so generated by Vinson[3] and Sterret and Barber[4] with experimental results also given in Fig. 1. A second approach, derived from Abdelhamid and Dosanjh,[5] is also given in Fig. 1. The first two solutions differ from the experimental data only by a proportionality constant, and there seems little justification at this point for the added complications advocated in Ref. 5. The exact cause for the observed difference between the theories and the experimental data of Fig. 1 is virtually impossible to isolate among the multitude of possibilities, but there seems little doubt that it is because of differences in detail and not in concept.

Since the two-dimensional experimental data of Fig. 1 was for air expanded through sonic nozzles, the influences of the specific heat ratio γ_e and the jet exit Mach number, M_e, were in question. However, Vinson[3] had used his characteristic solutions to show that there was no appreciable specific heat ratio influence for $1.2 < \gamma_e < 1.4$, but that there was strong dependence on the jet exit Mach number. Although there was no two-dimensional experimental evidence to contradict Vinson's results, it was felt this point needed further study. An attempt was therefore made to use the large reservoir of axisymmetric freejet data to supplement the two-dimensional sonic data. The vehicle for this effort was to be the

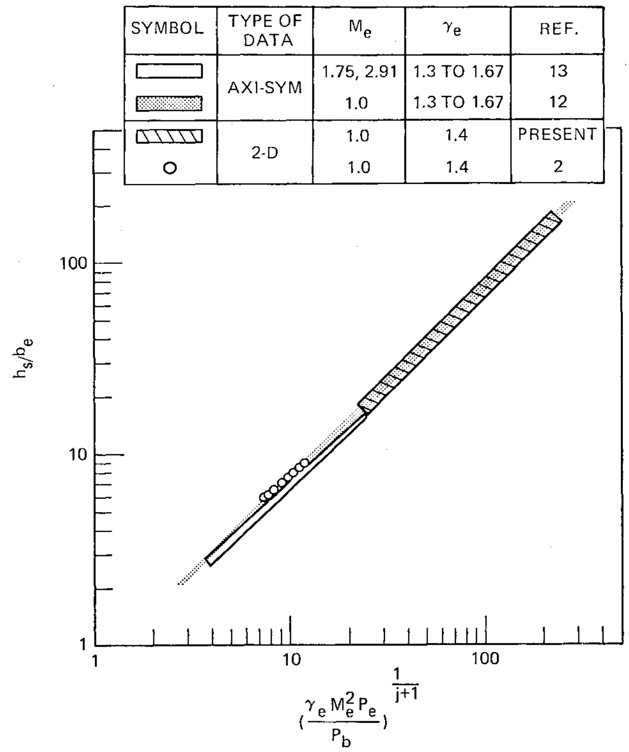

Fig. 2 Universal correlation of freejet experimental data.

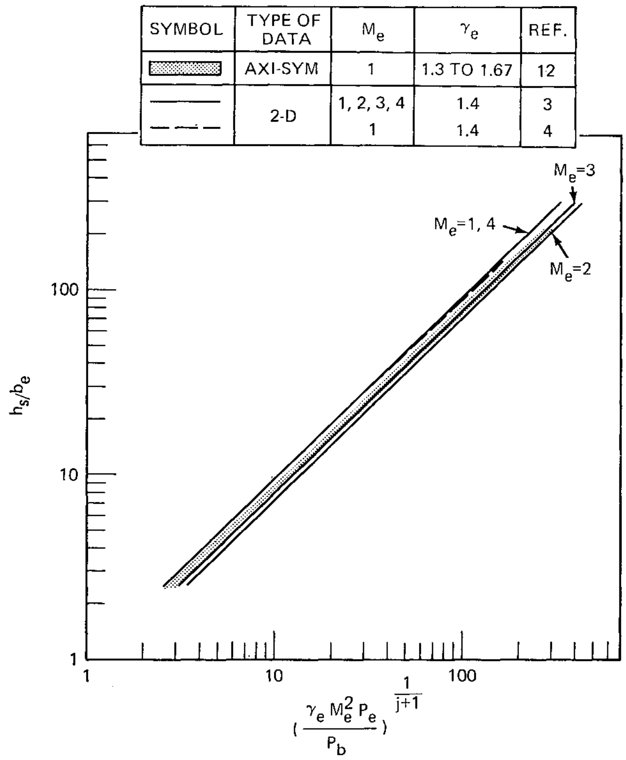

Fig. 3 Universal correlation of freejet characteristics solutions.

classical freejet model¶ of Mirels and Mullen,[6] which is supposedly valid for both the axisymmetric and two-dimensional case. Even though the model used in that formulation is only valid for hypersonic jet Mach numbers, it is a simple matter to show that the model should apply equally well to the far field of a freejet. From such a model though, the best one can obtain is that the density along the jet centerline ρ_{cl} decays like

$$\rho_{cl}/\rho_e = A(x/b_e)^{-(j+1)} \tag{2}$$

where $j = 0$ for two-dimensional flow and $j = 1$ for the axisymmetric case. References 6–11 contain arguments for determining the dependence of A in Eq. (2) on the various parameters of the problem. Unfortunately, none of the proposed methods adequately correlated two-dimensional and axisymmetric terminal shock data—it being found that not only were these models in error quantitatively, but they also suffered the serious qualitative fault of giving a wrong trend as one went from $j = 0$ to $j = 1$. Further comparisons showed this freejet model to be oversensitive to jet exit Mach number variations in comparison to the experimental data, a result well typified by the comparison given in Bauer's Fig. 2.[11]

Ideally, it would be hoped that at this point in time, the introduction of the new two-dimensional data reported herein would allow a "tuning up" of the analytical model. However, such an approach seemed inappropriate since some of the difficulties encountered to date undoubtedly find their basis in conceptual problems associated with the flow modeling. For example, how should one account for the barrel shock observed in all experimental freejet plumes?

One point that has not as yet been exploited is that the nondimensional governing equations quite naturally give jet exit dynamic pressure as the characteristic pressure for nondimensional purposes. Combining this with Eq. (2) leads to the supposition that

$$h_s/b_e \; \alpha \; (P_b/\gamma_e M_e^2 P_e)^{-(1/j+1)} \tag{3}$$

¶ Significant updates to this model have been given by Wedemeyer,[7] Hubbard,[8] Hill and Draper,[9] Phinney,[10] and Bauer.[11]

Application of this correlation to available freejet data is shown in Fig. 2 where the proportionality constant is found to be approximately 0.7 for all cases. This final form of the correlation appears to be a direct extension of Lewis and Carlson's[13] axisymmetric equation.

Equation (3) seemingly provides the sought after generalization of the two-dimensional sonic air jet correlation [Eq. (1)]. Further evidence to support this last point can be found by comparing the characteristics based solutions with the predictions of Eq. (3), keeping in mind that (as shown in Fig. 1) such solutions were only in error by a proportionality constant for the sonic case. Figure 3 shows that such a comparison for a wide range of two-dimensional exit Mach numbers and axisymmetric exit specific heat ratios is quite encouraging. In general, these values do differ only by a proportionality constant from the experimental values of Fig. 2. However, there is one perplexing failure evident when one considers Vinson's[3] results for other than $\gamma_e = 1.4$. Vinson found that variations in the exit specific heat ratio produced virtually no variation in the predicted shock height h_s/b_e. Thus, plotting any of Vinson's results for $\gamma_e \neq 1.4$ on the scale of Fig. 3 would cause a proportional shift in the curve labeled for Ref. 3. At present there is no reasonable explanation for this discrepancy. It is evident, though, that the correlation of Fig. 2 stands alone quite well and is not significantly faulted by the lack of complete comparison with characteristic solutions.

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Reducing Ballistic Range Data for the Projectile's Drag Coefficient

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THE ballistic range has been used extensively to determine the drag coefficient of spherical projectiles.[1–3] The usual practice in data reduction has been to fit the measured times by a least-squares technique to a second- or higher-order polynominal in distance and relate the coefficients thereby obtained to the drag coefficient. In flow regimes where the drag coefficient does not vary appreciably with velocity, such as hypersonic flow, the polynominal-fit technique imposes excessive precision requirements on the measurements. It is more practical in these flow regimes to use the time-distance relation for a projectile with a constant drag coefficient and determine that value which provides the best fit with the data.

The momentum equation describing the deceleration of a projectile in a stagnant atmosphere is

$$m dV/dt = -\rho V^2 C_D A/2 \tag{1}$$

Rewriting

$$dV/dt = V dV/ds$$

and integrating the previous equation, assuming C_D is constant, yields

$$V = V_0 \exp(-\rho C_D A s/2m) \tag{2}$$

where $V = V_0$ when $s = 0$. Expressing velocity as the time derivative of distance and integrating once more gives

$$t = t_0 + [\exp(\rho C_D A s/2m) - 1]/(V_0 \rho C_D A/2m) \tag{3}$$

where t_0 is the time when $s = 0$. Nondimensionalizing this equation with respect to range distance s_R and traverse time for the projectile t_R, results in

$$\bar{t} = \bar{t}_0 + (e^{\alpha \bar{s}} - 1)/\alpha \bar{V}_0 \tag{4}$$

where

$$\alpha = \rho A C_D s_R/2m$$

and the bar designates a nondimensional quantity. Equation (4) is the relation to be used for the best fit of the time-distance data.

Two data-reduction schemes based on the preceding equations have been proposed. Seiff,[4] having noted the relationship expressed by Eq. (2), suggested that the average velocity between each pair of measuring stations be calculated and plotted vs distance on semilog coordinates. The slope of the line passing through the data points is proportional to the drag coefficient. The difficulty with this scheme is locating that point in the interval where the velocity is equal to the average velocity. Seiff recommends first assuming that the average velocity occurs at the interval's midpoint and then altering this location, using the calculated drag coefficient, until the best fit with a straight line is obtained. Obviously errors can be introduced using this computational and graphical procedure, especially if large velocity decays occur between stations.

Pitkin,[5] utilizing the relation expressed by Eq. (4), proposed that the measured times corresponding to a pair of

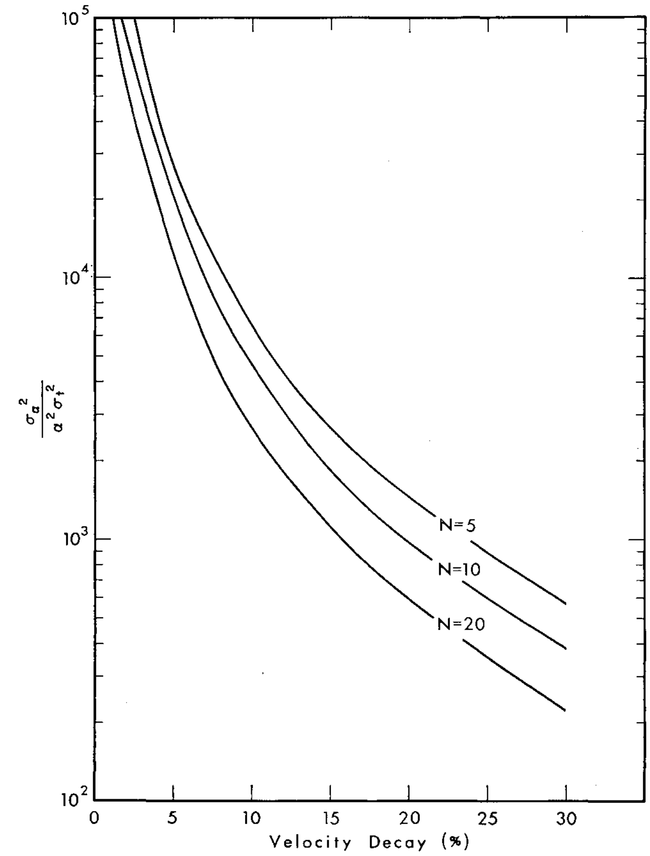

Fig. 1 Dependence of $\sigma_\alpha{}^2/\alpha^2\sigma_t{}^2$ on velocity decay and number of measuring stations.

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